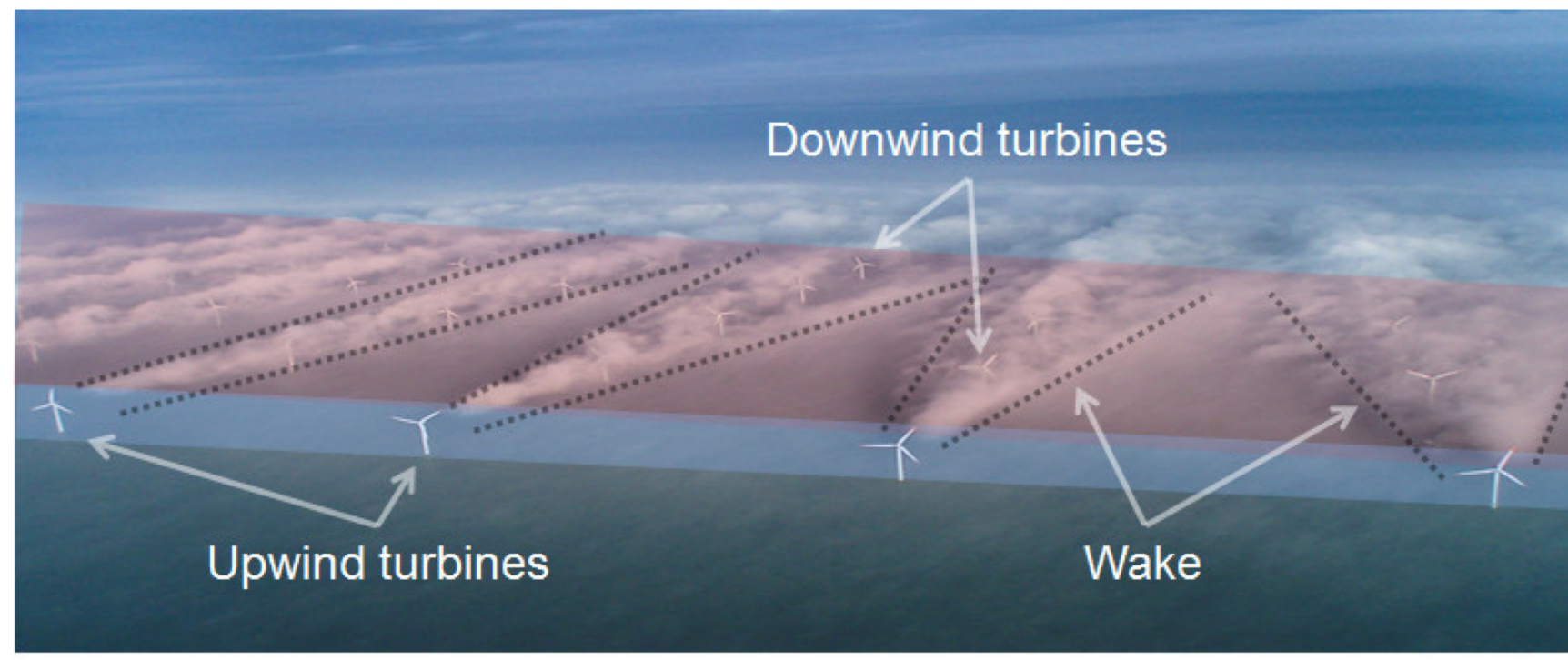


Problem



In the wind farms downstream turbines' efficiency is lower due to the **wake effect**.

Jensen's wake model introduces unknown parameters like the wake decay coefficient (θ).

Parameter calibration aims to find a suitable parameter value, well-informed by data uncertainty.

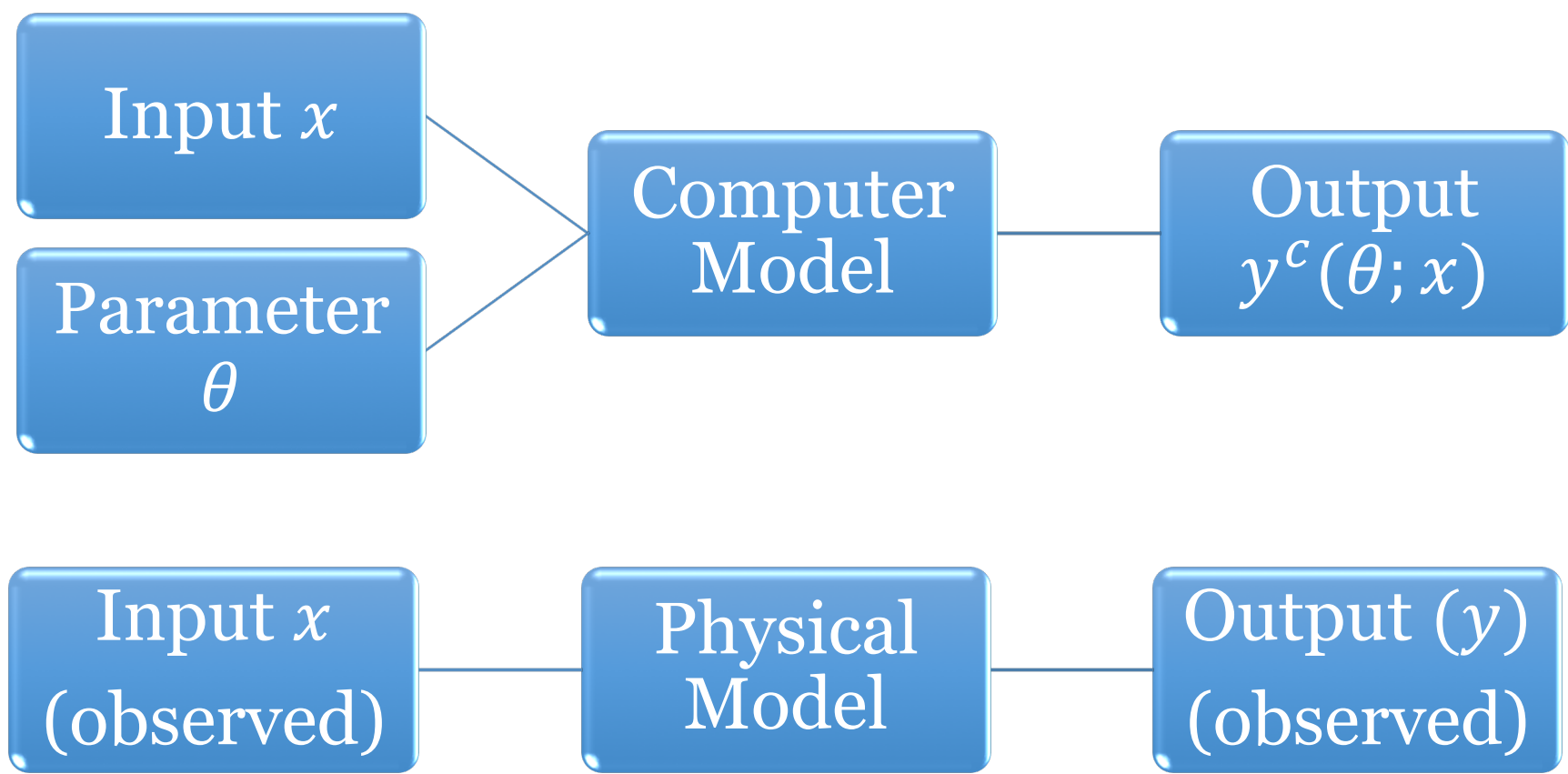


Fig 1: Computer model parameter calibration.

- In the literature, Bayesian calibration has been widely used, but it has limitations in handling large scale data.
- We cast calibration problem with stochastic optimization algorithms to handle big data.

Data-driven Stochastic Optimization

Suppose we have $\langle x_j, y_j \rangle j = 1, \dots, n$ data points, where x_j is some characteristic of the wind (speed, intensity) and y_j is the observed generated power from the turbines. We seek

$$\min_{\theta \in [\theta_{\min}, \theta_{\max}]} f(\theta) := \mathbb{E}_{X,Y}[\ell(y^c(\theta; X), Y)].$$

The loss function is $\ell(y^c(\theta; X), Y) := \|y^c(\theta; X) - Y\|_1$.

Estimated objective is

$$F(\theta, \mathcal{S}(\theta)) = \frac{1}{|\mathcal{S}(\theta)|} \sum_{\langle x_j, y_j \rangle \in \mathcal{S}(\theta)} \ell(y^c(\theta; x_j), y_j).$$

Optimization is **derivative-free** with the main assumptions:

- $F(\theta, \mathcal{S}(\theta)) \xrightarrow{n \rightarrow \infty} f(\theta)$ almost surely for all θ .
- $f(\theta)$ is bounded below and continuously differentiable with Lipschitz continuous gradients for all θ .
- The parameter θ is independent of x_j 's.

Methodology

We use **Trust-region**, an iterative method that approximates $f(\theta)$ by

- a local model that suggests the next step in a Δ_k -neighborhood around current best solution θ_k ;
- its derivative-free version builds model on several θ 's near θ_k .

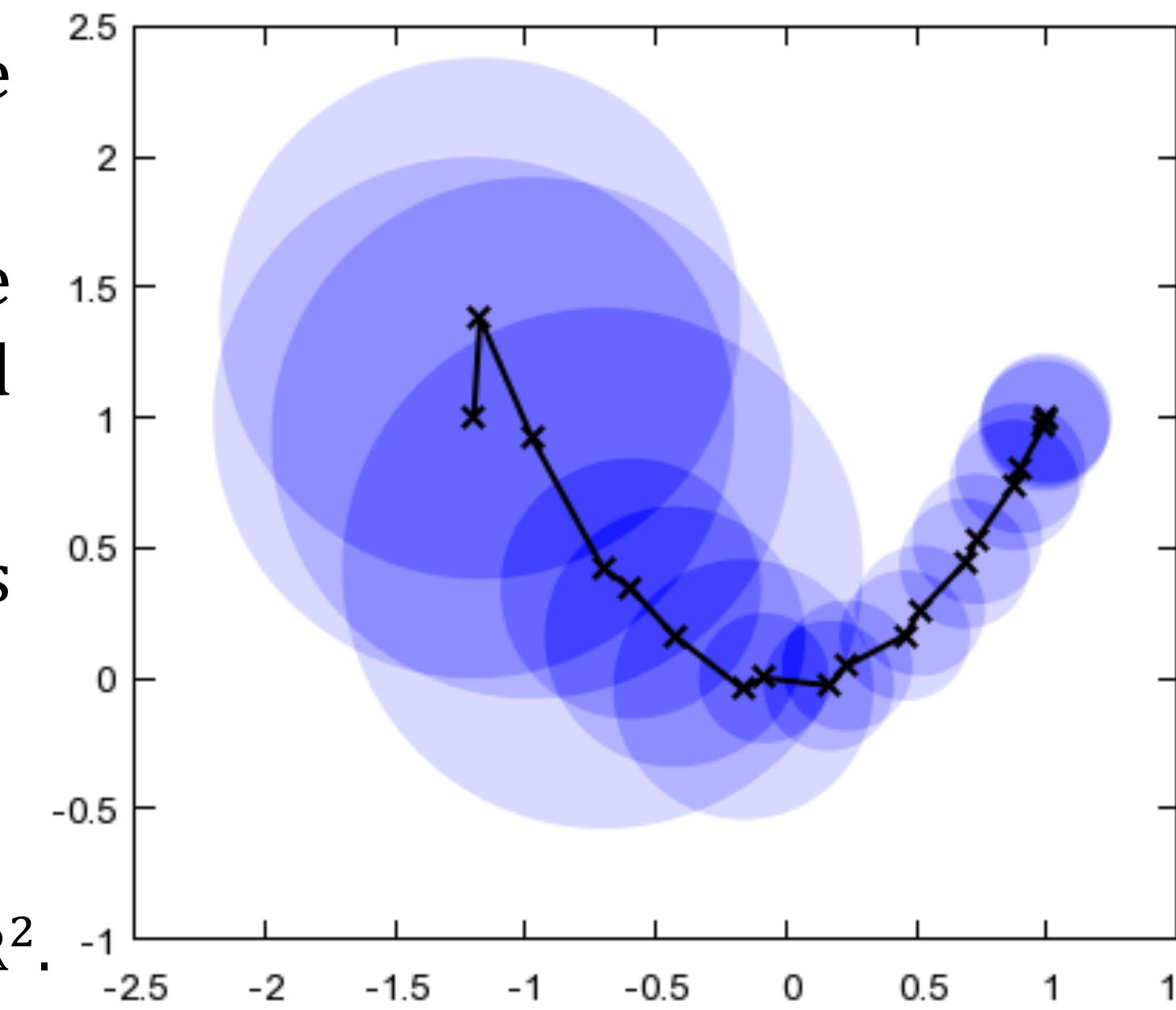


Fig 2: Trust-region optimization in \mathbb{R}^2 .

Benefits over Bayesian Calibration:

- response surface over f instead of Y is faster and has more details,
- models are iteratively updated,
- search direction is driven by updating the trust-region and model.

Adaptive sampling

- use a subset of data at each iteration to relieve computational burden,
- determine sample size adaptively at each solution; more samples if estimation error large relative to the optimization error.

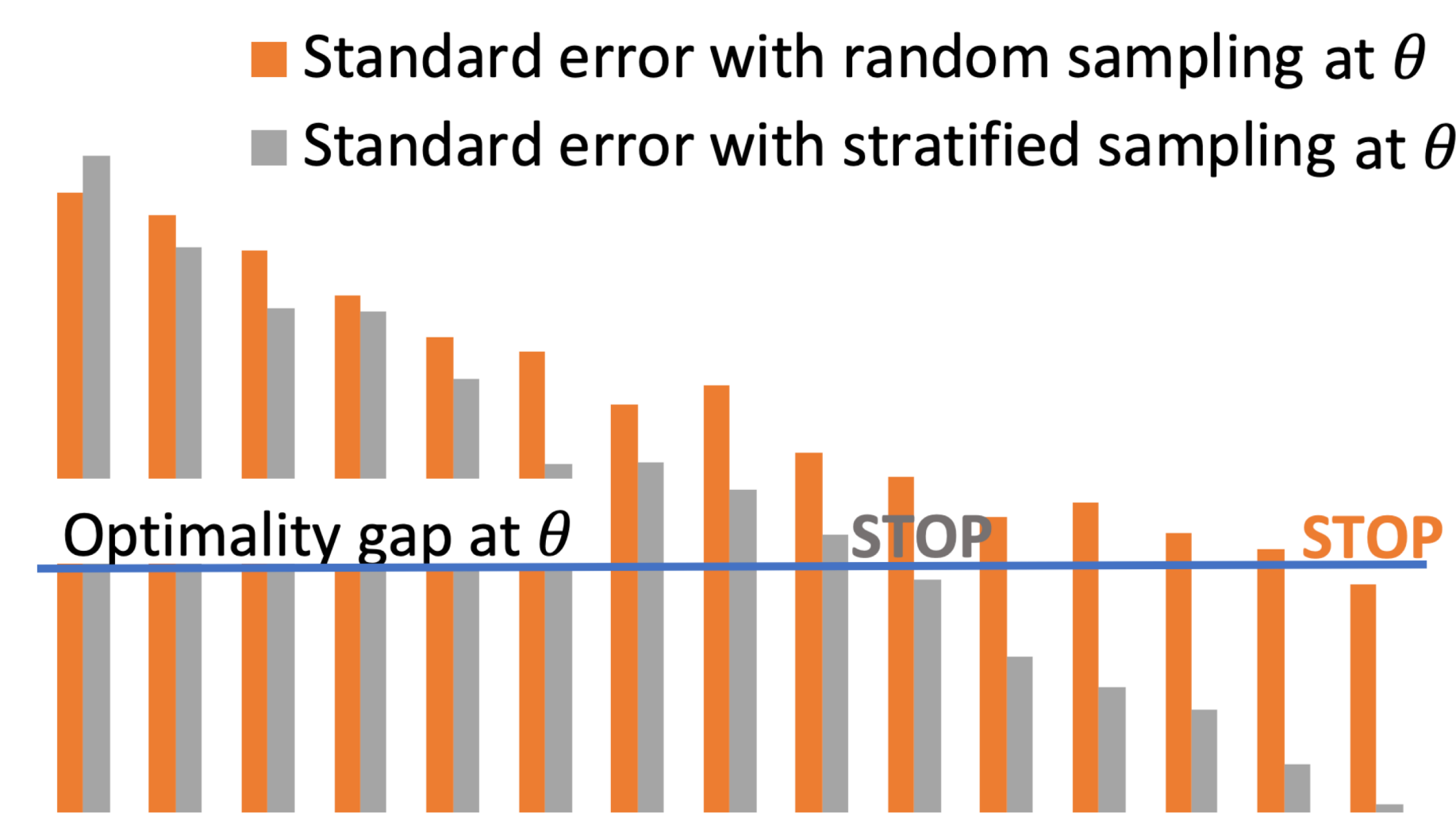


Fig 3: Error changes with sample size increase and stopping of adaptive sampling.

Stratified sampling

- further reduce the variance by allocating portions of the total points to the strata with higher variance.

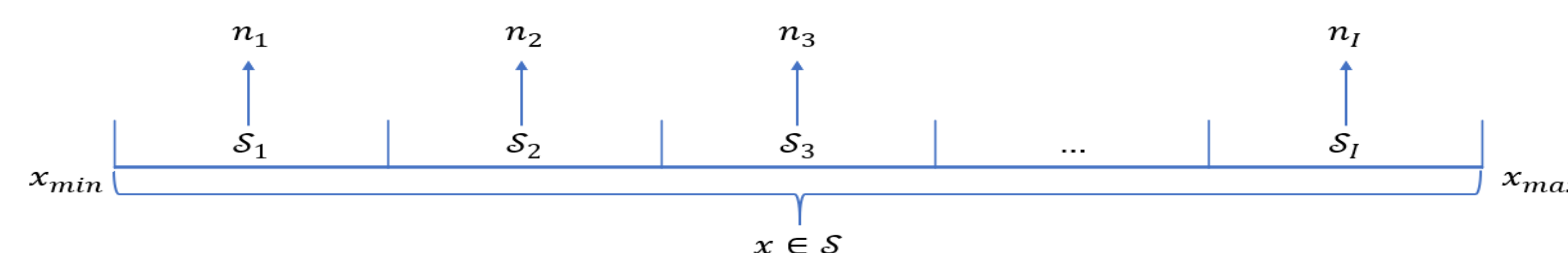
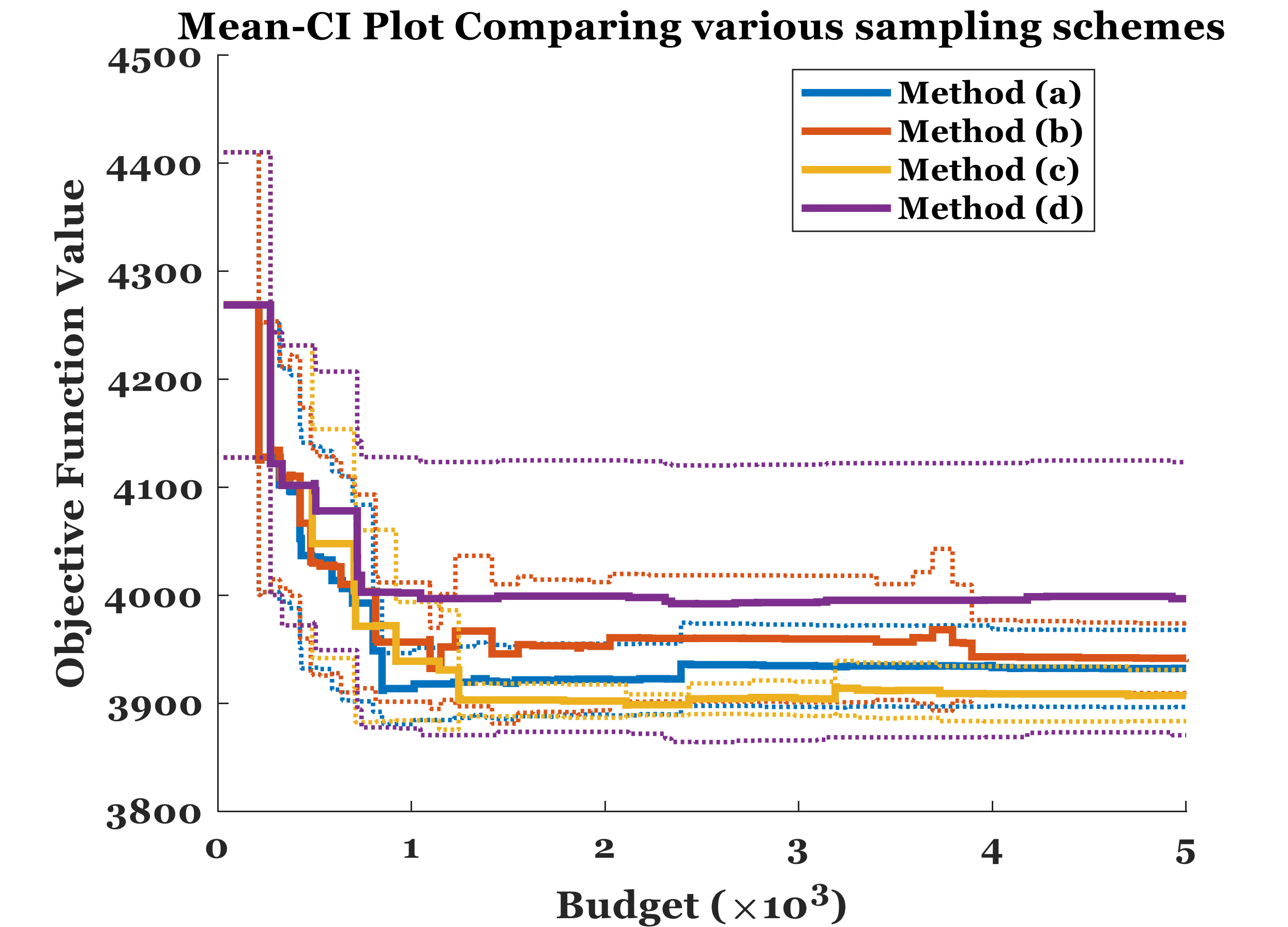


Fig 4: Stratified sampling using a one-dimensional variable.

- Sample size of stratum i : $n_i(\theta) := \lceil w_i(\theta) \times n(\theta) \rceil$.
- Weight of stratum i : $w_i(\theta) = \frac{p_i \hat{\sigma}_i(\theta)}{\sum_{i=1}^l p_i \hat{\sigma}_i(\theta)}$. What $\hat{\sigma}_i(\theta)$ to use here?

Experimental Results



(a): Weights via output variance $\hat{\sigma}_Y, \Delta n = 1$	(c): Weights via output variance $\hat{\sigma}_Y, \Delta n = b$
(b): Weights via loss variance $\hat{\sigma}_F, \Delta n = 1$	(d): Weights via loss variance $\hat{\sigma}_F, \Delta n = b$

Table 1: 4 sampling schemes determine how to sample points using adaptive and stratified sampling.

- Weights via $\hat{\sigma}_Y$ lead to higher accuracy compared to via $\hat{\sigma}_F$.
- Increasing samples in batches can be more efficient.

Conclusions

- Compared to Bayesian approach, trust-region approach builds local model. Construction of local models are driven via optimization structure. Thus, it can capture the local variations better, leading to effective parameter calibration.
- Combination of adaptive and stratified sampling gives robust solutions with lower variability.

References

- Bingjie Liu, Matthew Plumlee, and Eunshin Byon. Data-driven parameter calibration in wake models. In 2018 Wind Energy Symposium, 2018.
- Sara Shashaani, Fatemeh S Hashemi, and Raghu Pasupathy. ASTRO-DF: A class of adaptive sampling trust-region algorithms for derivative-free stochastic optimization. SIAM Journal on Optimization, 28(4):3145–3176, 2018.